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CAKE COMPRESSIBILITY ANALYSIS AND EMPIRIC
CORRELATION EQUATION

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Cake compressibility is an important factor to describe thickening, dynamic membrane formation, filtration, centrifugation, squeezing. It is characteristic of compressible cakes to change their porosity starting from the upper boundary into the bulk under the effect of newly formed underlying layers and the change of hydrodynamic conditions inside the pores of a cake.

The problem of the cakes properties description is as follows: existing empiric equations can be used in the limited ranges of compression stress (p_s) (effective pressure) acting on the cake skeleton. The lack of a method for cake properties interpretation in a wide range of p_s changing (from p_s over 0 to $p_s \gg 0.1$ MPa) deprives us of the possibility of effective cake properties control and so we cannot assure high effectiveness of separation equipment.

The attempt was done for cake compressibility phenomenon analysis at the position of nonequilibrium thermodynamic. In this case the critical value of p_s can be considered as point of bifurcation when dependencies of cake properties vs. effective pressure are changed. For example it can take place when p_s changes from low p_s to moderate values of p_s or from moderate to high p_s .

So, empiric correlation equation of cake properties for wide range of p_s and cake compressibility phenomenon analysis at the position of thermodynamic system are done below.

As shown by practice, the use of the empiric equation of the form:

$$Y = A * p_s^B \quad (1)$$

and form

$$Y = a * (1 + p_s / p_{si})^c \quad (2)$$

to characterize compressible cake properties, i.e. cake specific resistance, α , solid phase volume fraction in a cake, ϵ_s , cake permeability, K , gives the most satisfactory agreement of approximated correlations with the experimental data in

the average pressure range $p_s = 1 \times 10^4 - 1 \times 10^6$ Pa. Here p_s is the effective pressure, acting on the cake skeleton. For the flocculated cakes and other easily deformed structures, however, there are two areas of property change which differ in nature. These areas are conventionally separated by p_{si} - value that characterizes some range of the effective pressures where a transition from one kind of rules to another takes place.

According to often used model cake properties do not change when $p_s < p_{si}$, i.e. they remain constant ($Y = \text{const}$), Fig.1, only when $p_s > p_{si}$ the process of cake packing takes place (equation 2) or in a more simplified way the change of cake properties is expressed by the exponential equation 1, (Tiller F., 1988).

As systematic data on the behaviour of the compressible cakes under the influence of low effective pressure, p_s , are not available, we'll try to make a qualitative analysis of the phenomenon.

First of all it ought to bear in mind that the knowledge of unloaded cake properties ($p_s = 0$) is relative, as such parameters as K and α can be estimated only in the result of movement of a fluid through a cake, therefore, condition $p_s = 0$ is not satisfied. K - estimates by Coseny-Carman equation based on ϵ and S_0 (specific surface area) give approximate values of α_0 , K_0 or ϵ_{s0} for $p_s = 0$, as the accuracy of the estimates of a specific particle surface area is not great and depends on many factors that are often uncontrollable.

Therefore, it is reasonable to speak about "conventional" cake parameters at $p_s = 0$ obtained by extrapolation of the experimental data.

When investigating compressible cake properties the conditions of cake formation should be taken into account. Regardless of cake formation on filtration (effect of forces of friction of a moving liquid), or settling under the action of gravitation force or electrostatic settling, cake is formed from the dispersed particles under the action of the effective pressure imposed on solid particle carcass. The order of the magnitude of p_s can change, but it differs from zero.

In some models a surface layer is regarded as an unloaded layer of the cake being formed, e.g. during filtration or settling. This approach has some shortcomings too.

When settling the boundary between a cake and a suspension is not distinct. There is a smooth transition from the properties

of a condensed suspension in the area closer to the cake properties for the fine dispersed suspensions. During filtration the particles experience dynamic loads even at the moment of their deposition onto the cake surface.

Despite all the complexity of the problem, the derivation of the qualitative models of the compressible cake behaviour is extremely important for the low p_s - range, as the great volumes of commercial suspensions with flocculated dispersed phase are processed in industry. The nature of properties change of such suspensions with low p_s - values often determines all the further procedure of the dewatering processes: by settling, filtration, electrostatic settling, etc.

Going back to the model of constant cake properties in the range of $p_s < p_{si}$ (eq.1) the following disadvantages should be noted: First of all there is a jump in cake properties at the point p_{si} , where $|dY/dp_s|$ changes from zero to maximum. This means instantaneous displacement of the particles at $p_s = p_{si}$ or instantaneous failure of an original structure, however, in real cakes there are gradual qualitative changes with p_s increase. Furthermore, model parameters at $p_s < p_{si}$ are not functionally related to the cake behaviour in the range of $p_s > p_{si}$ and are present in the equations 1 and 2 as the coefficients A and a only (scale coefficients).

Choosing an equation we'll originate from the fact that in the point p_{si} there should be a smooth transition from one rule of a cake deformation to another. For α and ϵ_s it is an increasing function and for K - it is a decreasing function. As a cake is an arranged system it behaves as non-Newtonian body under the influence of the effective stress, that can be characterized by dY/dp_s at point $p_s = 0$.

To cover all range of possible rheological properties of a cake, which is a decreasing function, the chosen equation should satisfy the condition: dY/dp_s can have any value from 0 to ∞ . As a result, a graphic interpretation of the equation to be chosen will be as shown in Fig.2, if equation 1 is taken as an approximate function for $p_s > p_{si}$ range (as mentioned before in many practical cases equation 1 is valid for this range).

For different kinds of the curve in range I shown in Fig.2, the following values of dY/dp_s at $p_s = 0$ are valid: curve 3 - $dY/dp_s = 0$; 4 - dY/dp_s - final positive value; 2 - $dY/dp_s = \infty$.

Let's assume that the change of a compressible cake parameter in range I is described by the function $Y = f_1(p_s)$, $p_s \leq p_{si}$, and in range II - by the function $y = f_2(p_s)$, $p_s > p_{si}$ respectively. For a smooth transition of one function into another the following conditions should be satisfied at the point $p_s = p_{si}$:

$$\begin{cases} f_1(p_{si}) = f_2(p_{si}), \\ df_1(p_{si})/dp_s = df_2(p_{si})/dp_s. \end{cases} \quad (3)$$

An analysis of various functional correlations has shown that the function given below satisfies the above conditions:

$$Y = f_1(p_s) = a \cdot \exp(b \cdot p_s^\gamma), \quad (4)$$

for which

$$dY/dp_s = a \cdot b \cdot \gamma \cdot (p_s^{\gamma-1}) \cdot \exp(b \cdot p_s^\gamma).$$

Thus, the point p_{si} can be conventionally regarded as a transition point from one mechanism of cake consolidation to another. In the range $p_s \sim p_{si}$ the contribution of the mechanisms that determine the nature of cake consolidation is approximately the same.

When $\gamma = 0$ $df_1(p_{si})/dp_s \neq df_2(p_{si})/dp_s$, function $Y = \text{const}$ when $p_s \leq p_{si}$, and model going in equation 1 under restriction $p_s \leq p_{si}$, $Y = \text{const}$.

The correlation between the cake characteristics in area I and II under effective pressure.

Using (3) and (4) let's establish the correlation between equation (4) and (1) terms in the point $p_s = p_{si}$.

$$\begin{cases} a \cdot \exp(b \cdot p_{si}^\gamma) = A \cdot p_{si}^B \\ a \cdot b \cdot \gamma \cdot p_{si}^{\gamma-1} \cdot \exp(b \cdot p_{si}^\gamma) = A \cdot B \cdot p_{si}^{B-1} \end{cases}$$

If p_{si} and B are known from experience, then

$$b = B / (\gamma * p_{si}^\gamma) \tag{5}$$

It is difficult to apply the eq. 5 as it requires the estimation of γ - value that can be found from experimental results by the successive approximation method or by generalization of the experimental data for various suspensions.

The coefficient (a) in the equation 4 can be represented as follows

$$a = A * p_{si}^B / \exp(B/\gamma) \tag{6}$$

In a particular case when $\gamma = 1$ we have

$$b = B/\gamma, \quad a = A * p_{si}^B / \exp(B) \tag{7}$$

When $\gamma = 0$ $df_1(p_{si})/dp_s = df_2(p_{si})/dp_s$, function $Y = \text{const}$ when $p_s \leq p_{si}$, and model going in equation 1 under restriction $p_s \leq p_{si}$, $Y = \text{const}$.

Passing from a generalized formulation to α and \mathcal{E}_s , we'll obtain a formula for the specific cake resistance

$$\begin{cases} \alpha = \alpha_0 * \exp(b * p_s^N) & , \quad p_s \leq p_{si} \\ \alpha = \alpha_i * p_s^n & , \quad p_s > p_{si} \end{cases} \tag{8}$$

where $\alpha_0 = \alpha_i * p_{si}^n / \exp(n/N)$, $b = n / (N * p_{si}^N)$.

For the parameter \mathcal{E}_s we have

$$\begin{cases} \mathcal{E}_s = \mathcal{E}_{s0} * \exp(b_1 * p_s^B) & , \quad p_s \leq p_{si} \\ \mathcal{E}_s = \mathcal{E}_i * p_s^\beta & , \quad p_s > p_{si} \end{cases} \tag{9}$$

where $\mathcal{E}_{s0} = \mathcal{E}_i * p_{si}^\beta / \exp(\beta / B)$ and $b_1 = \beta / (B * p_{si}^B)$.

With zero effective pressure, $p_s = 0$, $\alpha = \alpha_0$, $\mathcal{E} = \mathcal{E}_{s0}$, $K = K_0$, when $N > 0$ and $B > 0$.

Equation for K can be found from conditions (Tiller F., Yeh C., 1990):

$$\alpha * \epsilon_s * K = \alpha_0 * \epsilon_{s0} * K_0 = 1 \quad (10)$$

Used eq. (8) and (9) when $p_s < p_{si}$ we'll obtain a formula for the permeability as follows

$$K = \exp[-(bp_s^N + b_1 p_s^B)] / (\alpha_0 \epsilon_{s0}) \quad (11)$$

$$K_0 = \exp(n/N + \beta/B) / (\alpha_i * \epsilon_{si} * p_{si}^{(n+\beta)}) \quad (12)$$

Equation (11) can be represente as follows

$$K = K_0 \exp(-b_2 p_s^\Delta),$$

where b_2 and Δ is coefficients,

$$b_2 p_s^\Delta = bp_s^N + b_1 p_s^B \quad (13)$$

After some transformations

$$K_0 = K_i \exp(\delta/\Delta) / p_{si}^\delta \quad (14)$$

$$\delta/\Delta = n/N + \beta/B. \quad (15)$$

where $\delta = n + \beta$; $K_i = 1 / (\alpha_i \epsilon_{si})$.

Fig. 3, 4 shows the correlation equations for some experimental data.

Us we see if B and N values negative the parameters α_0 , ϵ_{s0} and K_0 can't be reach e.g. $dY/dp_s = \infty$. It is a shortcomin of this model.

So, the suggested equation for effective pressures range $p_s < p_{si}$ allow to get some qualitative evaluations of cake properties correlations when $p_s < p_{si}$ and when $p_s > p_{si}$. They also permit to extrapolate data to point $p_s \rightarrow 0$, when $N > 0$ and $B > 0$. However to confirm or to refuse these correlations it's necessary to carry out systematic research into cake properties in the range of $p_s < p_{si}$.

It should be admitted however that without general theoretical approach to the problem under consideration any empiric correlations are limited in their possibilities. That's why we suggest to discuss properties of compressible cake from the point of view of nonequilibrium thermodynamics.

The preconditions for that are the following: compressible cake can be considered as open thermodynamic system which has nonlinear equations of dynamics for description and inside of which the

cooperative movement of particles takes place when the range of parameters deviation from equilibrium reaches a critical value. As a result the system from unstable position passes to the stable position which is accompanied by generating of a more regular structure.

The processes in the cake under p_s action can be described as follows. Cake properties changing is developed on thermodynamic branch of solutions when small systematic deviations of processes from equilibrium take place. The appeared cake structure variations are obtained by continuous deformation of equilibrium structures and therefore have considerable likeness with them. But after some critical value of p_s the thermodynamic branch of solutions become unstable. In that case any small disturbance takes the system from thermodynamic branch to the new status which has more high structure regularity (bifurcation of solution). We can expect that near the critical point of p_s a new solution would be dependent from p_s nonanalytically which is the most typical for critical point in the region of high p_s .

In all likelihood research into the theory of compressible cake from the point of view of nonequilibrium thermodynamics may enrich substantially our ideas of mechanism of cake deformations formed in hydromechanical processes. The worked out empiric equations acquiring physical value.

Reference

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3. Tiller F.M., Yeh C.S. "Relative Liquid Removal in Filtration and Expression". Filtr. & Separ. (1990), 27, No 2, P.129-135.

Illustrations

Fig.1. Graphic representation of empiric models used to describe compressible cake properties. 1 - equation 1; 2 - equation 2; 3 - equation 1 for condition $p_s \ll p_{si}$, $Y=const$.

Fig.2. Graphic interpretation of the model to be chosen: I - $p_s \ll p_{si}$ range; II - $p_s \gg p_{si}$ range (equation 1).

1 - parameter change function in range II; 2-4 - parameter change function in range I.

Fig. 3. Data α vs. p_s (Grace, 1953) for
 FINE ILMENITE, $p_{si} = 6$ P.S.I., p_s), $p_{si} \alpha = 1.72 \times 10^{11} * p_s^{0.185}$;
 $p_s \alpha = 2.672 \times 10^{11} * \exp(-2.3588 * p_s^{-1.725})$.

Fig. 4. Data $\mathcal{E}_s = (1 - \mathcal{E})$ vs. p_s (Grace, 1953) for
 FINE ILMENITE, $p_{si} = 6$ P.S.I., p_s), $p_{si} \mathcal{E}_s = 0.333 * p_s^{0.0428}$;
 $p_s \mathcal{E}_s = 0.366 * \exp(-1.244 * p_s^{-2.36})$.

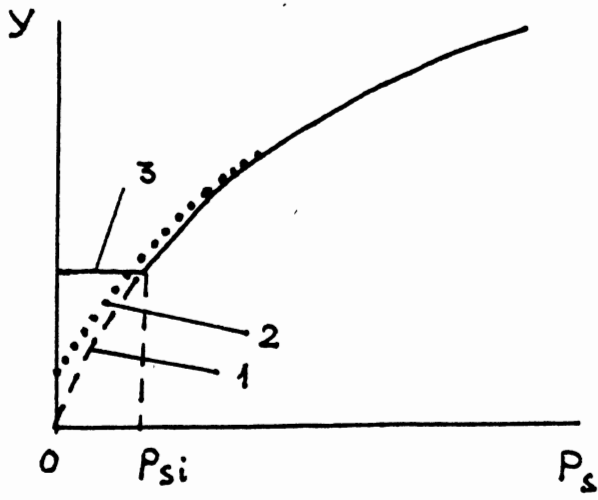


Fig. 1.

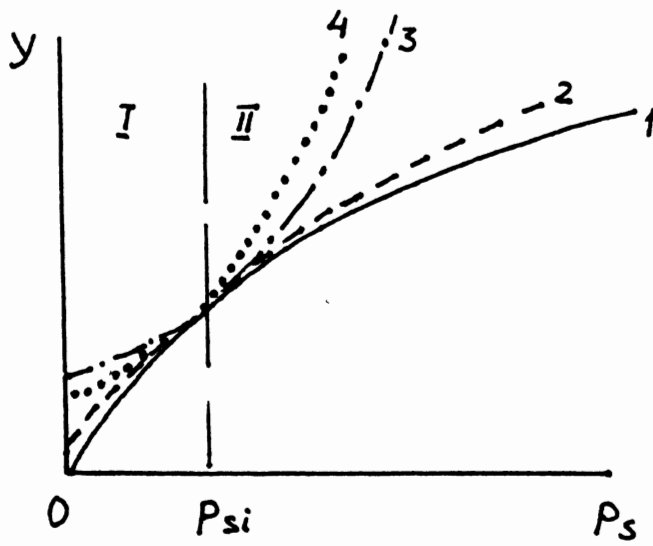


Fig. 2.

α , ft/Lb

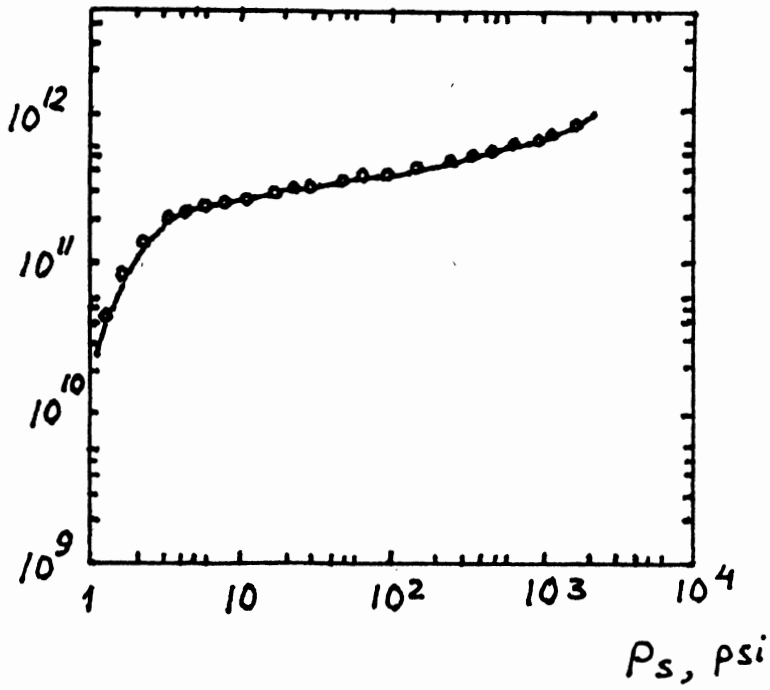


Fig. 3.

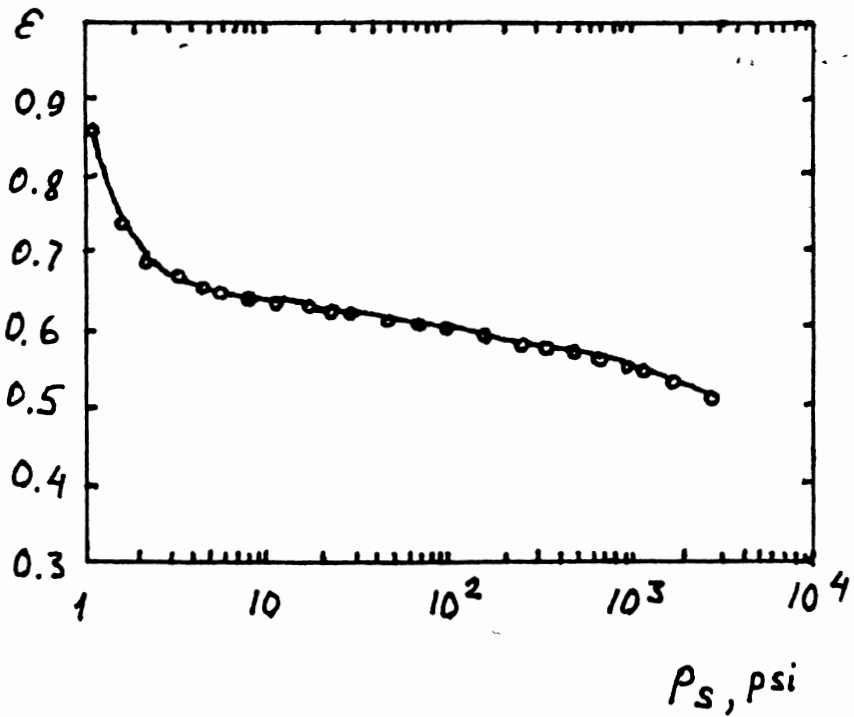


Fig. 4.