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On the possibility of using information entropy as a quantitative description of porous media structural characteristics

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Abstract

This paper deals with possibility of information entropy usage for porous media structure characteristics description. The paper presents the results of preliminary investigation of the possibility using information entropy parameter for porous media. The first approach of the method presented in the paper confirmed the possibility to get the join characteristic of porous media. The method may give a new point of view on the problem of porous media modelling. The examples of entropy calculation for distributions of pore by size and length as well as for multi-layers porous media joint entropy are given.

Keywords: Porous media; Characteristics; Information entropy

1. Introduction

The processes with porous media play an important role in nature and technique. The correct description of porous media is basic for the analysis and investigation phenomenon takes place inside them as well as the separation properties of the media: fluid dynamics [1]; adsorption [2]; chromatography [3]; membrane processes [4]. The existing porous media models abound with a large number of parameters and their application often presents a hard-to-solve problem [5].

Pore media characteristics on average approach the basis of the structure elements irregularity or regularity (it can be pore size, pore length or porosity distribution, etc.). Porous media integral properties: permeability, heat and mass transfer, etc., are derivatives of their joint structure irregularity [6–8]. Many researchers point out the joint effect of different media properties. This may be found in the majority of the articles mentioned herein. So, the problem of porous media description, analysis and prediction is still a task for applied science. New methods and equipment are involved in the investigations of this problem.

For instance, nuclear magnetic resonance spectroscopy and neural networks were used for gel pore size prediction based on data obtained from gel filtration chromatography and diffusion experiments [9]. The method seems too problematic for wide application because of complicated equipment and training neural network requirements. It has meant

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passing to other types of porous media for a new calibration of the method. The method of fractal geometry in combination with tomography and computer modelling [10–12] give new information about the materials structure.

Image analysis, 2D and 3D computer modelling [13,14] allows the possibility to obtain a multi-dimensional correlation of pore structure properties.

New experimental and theoretical data as well as a practical application of materials are of the barest necessity to elaborate the joint criteria for describing porous media integral properties. The ways to solve this problem do not come from a fractal approach, computer modelling or statistics.

By summing up fractal dimensions in two directions it is possible to obtain surface fractal dimensions [15,16]. It is possible by measuring the totality of modelling capillary pores with a uniform distribution upon the axis direction of the desired coordinate system to obtain some quantitative characteristics of porous media isotropy [17]. However, it is not possible with the methods mentioned to obtain a summarised value of different irregularity properties (size, length, porosity, etc.).

Two examples explain the situation. Very often in practice an expert must compare different porous media samples with each other in order to choose the appropriate porous media.

Example 1. Heterogeneous one layer porous media. The samples have different distributions of pore size, pore length, etc. These parameters determine the separation properties of the samples. The expert can use two ways: test or use experience + intuition. If an integral approach to the criteria, that show the general value of porous media irregularity, could be found the expert would be able to optimise the choice or build a new porous medium on the basis of the given data.

Example 2. Anisotropic or multi-layer porous media. The problem of multi-layer porous media characterisation is very real for anisotropic membranes as well as for multi-layers filters. BETA filter bags from Rosedal Products, for example, contained about 11 layers [18]. Anisotropic membranes can characterise 2-3-4 layers. In this case it is very difficult to use the average values of pore size, length, etc., which arise from the probable distribution of the characteristics of total media irregularity.

Some integral characteristics were developed from the thermodynamic theory basis that was adopted for dispersed systems liquid–solid and porous media.

2. Thermodynamic approach

Various thermodynamic models have been used for dispersed systems and separation processes. The concept of entropy and free energy have been used for developing criteria for the separation efficiency, a criterion of the separation process makes it possible to make an estimation of sediment porosity, etc. A brief overview of the research is given in [19] and [20].

Concept of the method of application. A very small solid particle is treated simply as if it was a large molecule. In this case it is possible to use the entropy S from molecular thermodynamics

$$S = k\ln(W),$$

where k is the Boltzmann constant that is transformed to the normalized coefficient and W is the value of multiplicity.

The problem of the methodical application of this approach arises for non-mono-sized particle fractions and in the case of multi-fraction mixtures of particles. The idea developed with sedimentation [21]. The model operates on the porosity of the sediment and suspension and does not have particle size distribution and other parameters of porous media (sediment).

Nevertheless, the method gives some criteria by which to calculate the separation process efficiency. A more effective solution to the integral characteristic of porous media can be reached using the principle of information entropy and theory of information.

3. Information entropy

Information theory provides a quantitative mathematical description of systems designed to communicate or manipulate of information. It sets up a quantitative measure of information and of the capacity of various systems to transmit, store and otherwise process information [22,23,32]. Information in the context of this paper presents a particular choice of one type of description of a porous media ("message") from a set of possible alternate descriptions.

The information entropy is the expected value of this amount of information and can be considered to be the average information of the message set.

The information measure has the following main qualities [24]:

1. The information in separate, independent outcomes should be additive.
2. Information should be proportional to the uncertainty of the source outcome.
3. The quantity of information should relate to the number of symbols needed to define the outcome.

The measure that satisfies these three is the expression $\ln[1/P(X)]$ with X the outcome of the source and $P(X)$ its probability.

If a physical system of random variables has different results then it posses some uncertainty. The measure of the indefinite system can be a value of information entropy H. According to the theory, if we have a variety X, then for every event $x_i \in X$ the entropy corresponds

$$H(x_i) = -\ln[p(x_i)], \quad (1)$$

which is the quantitative measure of the event uncertainty. The quantity of uncertainty for all totality of random events is averaged as follows

$$H(X) = -\sum_{i=1}^{n} p(x_i)\ln[p(x_i)], \quad (2)$$

where $p(x_i)$ is the probability associated with symbol i. The least possible information corresponds to maximum entropy, that is, when $p_1 = p_2 = ... = p_n = 1/n$.

The joint entropy of n independent random values is

$$H_{\Sigma}(X) = H_1(X) + H_2(X) + ... = \sum_{1}^{n} H_i(X), \quad (3)$$

In the most simple case the amount of information is $I(X) = -H(X)$. Usually the information I is present as a difference between the entropy of the outcome "message" and the entropy of the "message" after reception.

As pointed out [25] the correlation between entropy and information resemble in some sense the correlation between the physics notion of the potential and the difference of potentials. Entropy is an absolute measure of information and information is related to the definite change in experimental conditions.

The information approach has some limitations. The information determines the probable properties of events only, as the value that has no variety does not have information.

The information theory was developed as a theory of determining the quantity of sending and receiving information through a communication channel and for other applications, of course, needs additional criteria of information quantity and quality description. In this paper attention will be paid to the information entropy properties of porous media.

4. Application of information approach in applied science

The information approach is used for analysis of processes: rectification, absorption, adsorption, fluid flow, mixing of bulk materials, etc. [26]. Usually these were binary systems. The principle of the analysis was named the information-entropy method but its interpretation of information entropy is closer to thermodynamics than to information.

Every stream in a technological process can be considered as a stream of information and the Gibbs' potential can be used for the investigation of streams [27]. The point of count out of solving the information task was the condition $H = 0$ and the initial conditions of the process streams was $H > 0$. Processing can be considered as reducing the uncertainty by means of doing work and receiving information $I = -H$.

Assuming the processes are irreversible:

$$I_n = I_0 - \Delta I_n,$$

where I_n is the quantity of information in an irreversible process, I_0 is the quantity of information in a reversible process and ΔI_0–are information losses at the expense of the process irreversibility.

Information can presented by the equation

$$I = \sum_{i=1}^{k} p_i \ln(p_i), \quad (4)$$

where k is the number of states that information stream can take as a result of transformation in the chemical-technological process, $p_i = N_i/N$ is the probability of the i-state of the information stream.

The information state before and after transformation evaluated by the Gibbs' potential Z under the conditions of total stream energy constancy:

$$Z = f(p;t;N_1;N_2;...;N_k),$$

where p is the pressure and t the temperature. To the state N_i corresponds the value ΔZ_i of the potential changing in the technological stream

$$N_i = N\exp(-\Delta Z_i/RT).$$

The potential counted out from the "zero" level Z_0. It means for all other states $Z_i > 0$. Hence

$$p_i = N_i/N = \exp(-\Delta Z_i/RT).$$

Substitution of p_i in Eq. (4) gives

$$I_0 = \sum_{i=1}^{k} [-(\Delta Z_i/RT)\exp(-\Delta Z_i/RT)].$$

The value of the process information perfection evaluated through the information coefficient of efficiency is

$$\eta_{\text{inf}} = I_n/I_0 = 1 - \Delta I_n/I_0, \ 1 \geq \eta_{\text{inf}} \geq 0.$$

The following equation resulted:

$$H_{\text{output}} = H_{\text{input}}(1 - \eta_{\text{inf}}).$$

The method described above does not give objective models and criteria for systems such as porous media since it does not take into account the structure elements, for instance size distribution, etc.

Theory of information for biological membranes. The information principles used for the description of transport phenomenon in biological membranes was formulated as follows [28]: the membrane serves for information transfer (ions and molecules transfer through the membrane channels), moreover the vital importance has quality (cost) not quantity of information. The quality (cost) of information may be determined in the result analysis of the received message only. The quantity and cost of information may change in determined cases in opposite directions. Biological membranes do not transfer information only but serve as a selector of information quality.

If we move from the position of process analysis such as the information stream, to examine processes such as information channels (communication for information transfer), it is possible to find some new approaches for modelling processes with porous media.

5. Application of information concept to porous media

The information theory application to porous media description has been debated [29–31]. Below the value of the information entropy will be used as the porous structure irregularity measure.

The structure elements totality can be present as a random or a continuous probabilistic ensemble that can be determined for a discrete variable as a multitude X with a given probability distribution $p(x)\{X, p(x)\}$. For a continuous variable the ensemble can be present as $\{X, f(x)\}$, where $f(x)$ is the probability density function.

The information entropy for continuous distribution with a probability density function $f(x)$ can be present as follows [32]:

$$H(X) = -\int_{-\infty}^{\infty} f(x)\ln[f(x)]\,dx. \tag{5}$$

The value of $H(X)$ in this case, the opposite discrete variable, Eq. (2), may be negative and may become infinitely large.

It is this property that makes the information entropy different from the thermodynamics entropy. However, the information entropy values can be compared with each other:

$$\Delta H = H_1(X) - H_0(X), \tag{6}$$

where $H_0(X)$ and $H_1(X)$ are the values of entropy for two different $f(x)$.

5.1. Pore size distribution

A simple type of variable X distribution (Simpson distribution) can be considered for clarity:

$$f(x) = \begin{cases} 2h(x-a)/(b-a), & x \in [a, m_x] \\ 2h(b-x)/(b-a), & x \in [m_x, b] \end{cases} \tag{7}$$

$$H(X) = -\ln(h) + 0.5 = \ln\left[\frac{(b-a)\sqrt{e}}{2}\right] = \ln\left[(b-m_x)\sqrt{e}\right], \tag{8}$$

where $m_x = (a+b)/2$ and $h = 2/(b-a)$ are the average of x and the maximum of $f(x)$, respectively.

If we take into consideration that x is, for example, the membrane pore size, a and b are the maximum and minimum pore size ($x_{min} = a, x_{max} = b$), accordingly, and if $H(X)$ is interpreted as the measure of membrane pore irregularity in terms of pore size, the following qualitative conclusions on $H(X)$ can be done: the degree of structure element irregularity is decreased with decrease of pore size variation range $(b-a)$. The $H(X)$ follows it.

The possibility of comparison arises for porous media, with different ranges of structure elements variation within a definite distribution $f(x)$, if information entropy is taken as a measure of structure irregularity of elements X.

The shortcomings of the method discussed is the ideal porous media model with identical size of all pores that cannot be described here because $\Delta = b - a \to 0, H(X) \to -\infty$. However in real objects an ideal porous media model is not realised because of negligible but finite deviation of the random variable X from the average value m_x. When the porous media is approaching the ideal model the inequality $m_x - \delta \leq x \leq m_x + \delta$ always takes place. Here δ has a finite small value, and $H(X)$ has a finite value also.

Example. For even pore size distribution the average pore size $m_x = (a+b)/2$, then $H(X) = \ln[2(m_x - a)] = \ln[2(b - m_x)]$. If pore size x has a deviation from the average value m_x in the range $\pm 0.01 m_x$, the entropy value is $H(X) = \ln(0.02 m_x) = -3.94$ (for convenience $m_x = 1$). For deviation $\pm 0.001 m_x$ and $\pm 0.0001 m_x$ the value of $H(X) = -6.21$ and $H(X) = -8.517$. As we can see, even for a small deviation of pore size from the average value, $H(X)$ is relatively not far from the point $H(X) = 0$. The same is expected, for instance, for a normal distribution $x \in (-\infty, \infty)$, $H(X) = \ln(\sigma\sqrt{2\pi e})$, where σ is the standard deviation.

Consider a more general case for different types of random variable in the range $x > 0$ and put a list of information entropy as follows (expressions of $f(x)$ are given in Appendix A):

(1) *Exponential distribution*

$$H(X) = \ln(e/\lambda) = \ln(em_x) = 1 + \ln(m_x)$$

$$m_x = 1/\lambda$$

$m_x = EX$ is the expectation of the variable.

(2) *Gamma distribution*

$$H(X) = \ln[\Gamma(\alpha)] - (\alpha - 1)\psi(\alpha) + \alpha - \ln(\lambda),$$

$$EX = \alpha/\lambda$$

where $\psi(\alpha)$ is the ψ-function,

$$\psi(\alpha) = \mathrm{d}[\ln(\Gamma(\alpha))]/\mathrm{d}\alpha.$$

(3) *Log normal distribution*

$$H(X) = \ln(\beta e^m \sqrt{2\pi e}) = \ln(\beta\sqrt{2\pi e}) + m$$

m is the expectation of $\ln(x)$.

(4) *Weibull distribution*

$$H(X) = 1 + \frac{\alpha - 1}{\alpha}(c + \ln(\lambda)) - \ln(\alpha\lambda)$$

$$c \approx 0.5772$$

As we can see, even for the same expectation of the variable, we have a different value of the information entropy for different distributions. On the other hand if the degree of irregularity $H(X)$ is given, the type of variable distribution $f(x)$ can be selected thus the desirable range of variable x changing is ensured.

Since $H(X)$ for continuous distributions of random variable is unlimited in value, in some cases for analysis it may be more suitable to introduce limitations of $H(X)$. The choice of limits depends on conditions used in the task analysis. For instance, in analysis/comparison of porous media the boundary conditions for the variable range are the following: (1) minimal: porous media should have minimal standard deviation σ_{min} or range of deviation δ_{min}; (2) maximal: porous media should have maximal standard deviation σ_{max} or range of deviation δ_{max}.

The task is different for porous media analysis within the desired range of the variable. In this case we have $H_{min}(X)$ and $H_{max}(X)$, hence we may use the following formula for the analysis:

$$H^* = \frac{H(X) - H_{min}(X)}{H_{max}(X) - H_{min}(X)} \quad (9)$$

if $H_{max}(X) > H(X) > H_{min}(X)$.

When the purpose of analysis is selection porous media with $H(X)$ more or less the established limit one side limitation can be used:

$$\begin{aligned}\Delta H &= H(X) - H_{\min}(X) \\ &= H_{\min}(X)[H(X)/H_{\min}(X) - 1], \\ \Delta H &= H_{\max}(X) - H(X) \\ &= H_{\max}(X)[1 - H(X)/H_{\max}(X)].\end{aligned} \quad (10)$$

The value of $H(X)$ depends on the scale of variable used (microns, mm, etc.). The parameter $x^* \in [0,1]$ may be used for unification:

$$x^* = \frac{x - x_{\min}}{x_{\max} - x_{\min}}. \quad (11)$$

6. Application of information entropy

6.1. Distribution of structural elements axes in length

Consider a heterogeneous medium with "fibrous" (fibre-shaped) or "channelled" channel-shaped elements. Such media may include some types of capillary-porous materials, foams, foamed polymers, membranes, filters, composite materials, etc. Abstracting from the type of the cross-section of elements, to a first approximation we can consider the distribution of their axes lines in lengths. Assume that the distribution of structural elements in length is specified by a set Y and examines this phenomenon on porous media with through pores.

6.2. The simple examples of distribution of porous media in lengths

Consider a one-dimensional variant of length distribution of non-intersecting through capillaries ignoring the existence of dead end and closed on one surface capillary. In this case the smallest capillary length is equal to the depth of the porous media h in the direction normal to the surface of mass transfer. Assume that the length of the capillary and its diameter are independent.

If in evaluating pore (capillary) distribution in length we specify the minimum possible or desired relationship of capillary axes length y from the ideal ($y = h$), then by analogy with pore distribution in sizes we may take the value $H_{\min}(Y)$ as a starting point which complies with the specified boundary conditions:

$$H_{\min}(Y) = \ln[h(k_{\min} - 1)], \quad (12)$$

where $k_{\min}$ is the minimal degree of system disorder for tortuosity and h the porous media thickness.

Let us take the simplest case, that of even distribution of a random value y:

$$\begin{aligned}H(Y) &= \ln[(b-h)] = \ln[h(b/h - 1)] \\ &= \ln[h(k_M - 1)],\end{aligned} \quad (13)$$

where b is the maximum capillary axis length; $b/h = k_M > 1$ is the greatest ratio of capillary axis length to layer depth (the greatest tortuosity). Hence

$$\Delta H = H(Y) - H_{\min}(Y) = \ln\left[\frac{(k_M - 1)}{(k_{\min} - 1)}\right]. \quad (14)$$

Here instead of k_M and $k_{\min}$ we may use the average coefficients of tortuosity $\langle k_1 \rangle$ from distributions.

In this case it is possible to evaluate numerically and qualitatively entropy properties of certain types of porous media and to select optimal value of tortuosity.

Suppose that for two porous media the equality $H(Y) = H(Y') =$ constant is to be satisfied. Then in the simplest case of even distribution of Simpson distribution we have:

$$h/h' = (k' - 1)/(k - 1), \quad (15)$$

where $k = \langle k_l \rangle$ and $k' = \langle k'_l \rangle$, k_l and k'_l are coefficients of pore tortuosity of two media, respectively.

For numerical analysis assume $h = 10$, $k' = 2$ then for all media having the same value $H(Y)$ and the same distribution law of the random value y the equality $h(k-1) = 10$ is to be satisfied, in other words if $H(Y) = 5$ the coefficient of tortuosity must be equal to 3; $H(Y) = 100, k = 1.1$, etc. As we see, the degrees of disorder of porous media greatly depend on pore tortuosity.

With all other things being equal it is necessary:

- to minimise $H(Y)$ to solve the transfer problem, i.e. to decrease pore layer depth and pore tortuosity;
- to solve the problem of deep bed filtration for ensuring catching of dispersed phase it is necessary to maximize $H(Y)$, i.e. increase the tortuosity. The qualitative conclusions do not disagree with practical results.

Consider for illustration some numerical examples. Let us estimate how information entropy increment is changing in some hypothetical porous media with Simpson pore tortuosity distribution or even distribution when increasing or decreasing factors $m = h/h'$ or $n = k/k'$ for cases:

1. The total layer depth h changes at the constant value of mean pore tortuosity $k =$ constant.
2. The coefficient of tortuosity k changes at fixed total layer depth $h =$ constant.

Let h' and k' denote the initial depth and the tortuosity coefficient of the porous medium, respectively. Entropy increment with the change of the layer depth by a factor m with is equal $h = mh'$ to:

$$\Delta H = H(Y) - H(Y') = \ln(h/h') = \ln(m),$$
$$k = k' = \text{constant}. \tag{16}$$

With the change of the tortuosity coefficient by a factor of n with $k = k'n$

$$\Delta H = \ln\left(\frac{k-1}{k'-1}\right) = \ln\left(\frac{nk'-1}{k'-1}\right),$$
$$h = h' = \text{constant}, nk' > 1. \tag{17}$$

The result of the calculations are presented in Fig. 1, where curve 1 corresponds to Eq. (16), the change of cake layer depth with the constant coefficient of tortuosity. Curves 2–5 illustrate the calculations by Eq. (17) with constant layer depth for $k' = 10$, 2, 1.5 and 1.1, respectively.

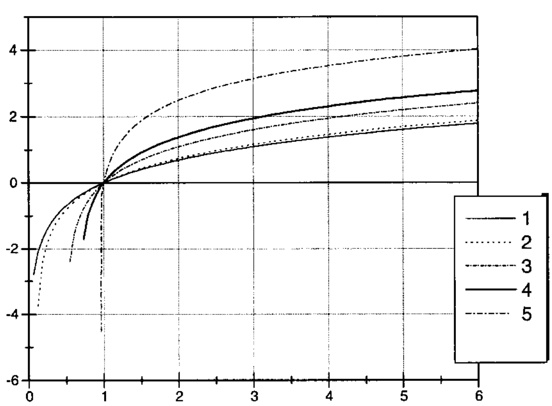

Fig. 1. Relation between ΔH and m,n. 1, ΔH vs. m with $k = k' =$ constant; 2—5, ΔH vs. n with $h = h' =$ constant for different k': 2, $k' = 10$; 3, $k' = 2.0$; 4, $k' = 1.5$; 5, $k' = 1.1$.

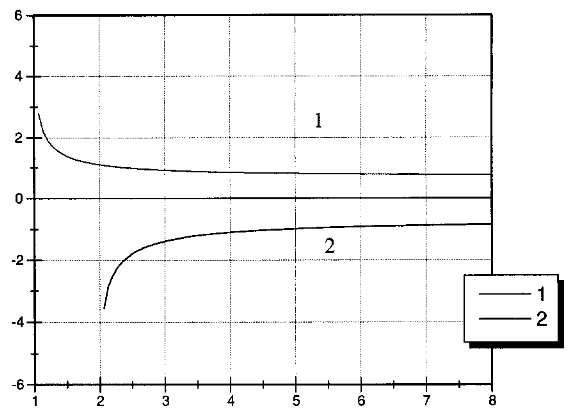

Fig. 2. ΔH vs. k' (see text).

Qualitative conclusions drawn from the obtained relations are consistent with practice. They are as follows:

- as regards the creation of a more ordered structure (improvement of transfer properties) of thin porous layers, especially membranes, depth decreases h with $k =$ constant is less effective than pore tortuosity coefficient decrease k with $h =$ constant;
- on the other hand, if k is close to 1, then even a little change of tortuosity in the direction of its increase results in multiple increment of information entropy ΔH, Fig. 1, curve 5 (the tortuosity may be change, for example, by means of porosity variation, changing type of particles packing or mixing particles of different size fractions);
- to increase retention in depth filters it is more effective to increase k with $h =$ constant, but with $k \gg 1$ this advantage is levelled as compared with the increase of h with $k =$ constant, and $\Delta H \to \ln(m)$.

The conclusions may be illustrated diagrammatically by Fig. 2, where curve 1 corresponds to the change of ΔH with $h =$ constant, $n = 2$, and curve 2 results from the same conditions but with $n = 1/2$.

In summery let us consider the condition of joint compensation of porous layer depth changing by factor of m with $h = h'm$ and tortuosity coefficient $k = k'n$:

$$\Delta H = \ln\left[\frac{m(nk' - 1)}{k' - 1}\right] = \text{constant} \tag{18}$$

For convenience let us take for simplifying the constant equal to zero and $k' = 2$. As a result we obtain a number of coupled relations between m and n, satisfying the condition of $\Delta H =$ constant.

m	10	5	3	2	1	1/2	1/4	1/5
n	11/20	3/5	2/3	3/4	1	1.5	2.5	3

In other words, if the producer of porous medium increases its depth by a factor m, it is possible to ensure the constancy of information entropy increment $\Delta H =$ constant as compared with a basic standard by way of appropriate correction of tortuosity coefficient value.

Condition (18) with $k' \gg 1$ goes over into approximate equation $\Delta H \approx \ln(mn) =$ constant.

Analogous conclusions may be reached for pore distribution in size as for example in the case of Simpson distribution:

$$\Delta H = H(X) - H(X') = \ln\left[\frac{m_x(\delta_x - 1)}{m'_x(\delta'_x - 1)}\right]$$

$$= \ln\left[\frac{m(n\delta'_x - 1)}{\delta'_x - 1}\right]. \tag{19}$$

where $m_x = \langle m \rangle$ is the average pore size, $\delta_x = b/m_x$ is the ration of the largest pore size to the average pore size; $m = m_x/m'_x$; $n = \delta_x/\delta'_x$.

6.3. Information entropy of multilayer porous media

Industry often use multilayer media with independent distribution of structural elements in layers, e.g. membrane with porous backing, multilayer filters.

Let us deal with the distribution of lengths of structural elements in multilayer media. We assume that structural elements of one layer pass into structural elements of the other layer. With permeable multilayer porous media it means continuity of capillaries through the whole volume of a porous layer, Fig. 3. As previously, the diameter and length of pores are assumed to be independent.

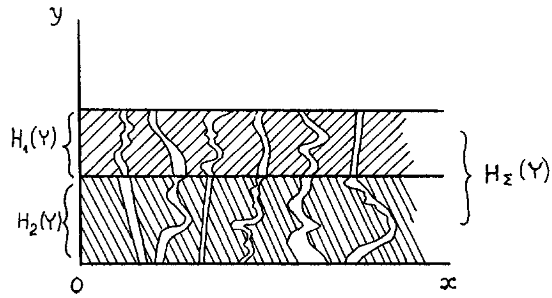

Fig. 3. Diagram of a two-layer porous material.

For multilayer media with independent parameters, joint information entropy is

$$H_\Sigma(Y) = H_1(Y) + H_2(Y) + \ldots + H_n(Y)$$

$$= \sum_{i=1}^{n} H_i(Y), \tag{20}$$

where $H_i(Y)$ is the mean information entropy (degree of disorder of i–layer structural elements).

The proposed method shows the way to monitor the summary value $H_\Sigma(Y)$ by changing the parameters of individual layers. It can be done during design and building porous media. Of course the entropy by itself does not give the solution and must be correlated with the properties of porous media.

Example 1. Suppose that for effective filtration in two-layer porous media. The first layer serves for capture of main part of particles. The second layer serves for retention control of most smallest particles passed through the first layer. Assume the system degree of disorder $H_\Sigma(Y)$ must be ≈ 3. Let us describe the pore distribution in the lengths of both layers by Simpson's distribution. Let us assume that the second filtering layer characterized by $H_2(Y) = -0.19$ that corresponds to $h_2 = 2$, $k_2 = 1.5$. To comply with the condition of $H_\Sigma(Y) \approx 3$ it is necessary to match such degree of disorder of the first layer.

Joint entropy of two independent random values is

$$H_\Sigma(Y) = H_1(Y) + H_2(Y), \tag{21}$$

whence

$$H_1(Y) = H_\Sigma(Y) - H_2(Y) = 3 - (-0.19) = 3.19.$$

Thus the degree of disorder of the first layer must be

$$H_1(Y) = \ln\left[\frac{\sqrt{e}}{2}h_1(k_1 - 1)\right] = 3.19.$$

Specifying the coefficient of tortuosity k_1 we obtain the depth of the first layer complying with the condition of $H_\Sigma(Y) \approx 3$:

$k_1 = 2$, $h_1 = 29.48$;
$k_1 = 3$, $h_1 = 14.74$;
$k_1 = 3.5$, $h_1 = 11.8$.

Thus if entropy characteristic of filtering medium is known and the value of joint entropy of the system "first + second layers" is specified we can select the parameters of a filtering material necessary to form the first layer (if of course the correlation between the entropy and filtration properties layer is known).

Qualitative results of the example show that selection of the first layer material with the greater coefficient of tortuosity k_1 allows for the layer with less depth. A great coefficient of tortuosity is characteristic of needle shape or fibrous filtering materials, and as it is known by experience with all other things being equal, the required depth of the first layer from fibrous material is less than the depth of the layer from granular material.

Example 2. Membrane with backing. Suppose that the membrane itself is characterized by information entropy $H_1(Y) = -0.89$ ($k_1 = 1.5$; $h_1 \approx 1$). It is necessary to match the porous backing which could provide the degree of disorder $H_\Sigma(Y) \leq 1$ for two-layer porous medium (membrane + backing). So, the required value of $H_2(Y)$ of the backing determined as

$$H_2(Y) = \ln\left[\frac{\sqrt{e}}{2}h(k_2 - 1)\right] = H_\Sigma(Y) - H_1(Y) = 1.89.$$

As a result if the backing has $k_2 = 1.75$, its depth h_2 is ≈ 10.7.

It is necessary to stress once again that the given results are qualitative, they do not supersede quantitative calculations. However the method developed gives the possibility in the future to correlate the transfer phenomena and the degree of disorder of structural elements of porous media.

6.4. Information entropy of independent random pore distribution in sizes and lengths

In case of independence of random values ensembles $\{f(x),X\}$ and $\{f(y),Y\}$ we can obtain joint information entropy $H(X,Y)$ by summation of $H(X)$ entropy of pore distribution in sizes and $H(Y)$ entropy of pore distribution in lengths:

$$H(X,Y) = H(X) + H(Y).$$

If the density of pore distribution in sizes $f(x)$ is characterized by beta-distribution and $f(y)$ by Simpson distribution, then

$$H(X,Y) = \ln[A(b-a)(k-1)h], \qquad (22)$$

where $A = \sqrt{e}/2.52$, b and a are the largest and the smallest pore sizes, respectively, k is the mean coefficient of tortuosity and h is the depth of the porous layer.

Let us analyse the relation obtained. As follows from Eq. (22) system disorder "pore length + pore diameter" decrease with narrowing of the interval $(b - a)$ and $(k - 1)$. It results in the decrease of pore size deviation from the mean value and pore length deviation from the layer depth h.

Depending on the function of the porous medium, the importance of pore size and length variations for the general criterion, i.e. joint information entropy $H(X,Y)$, will be different, and it can be taken into account by introducing the notion of information value or functional penalties. But this problem is not to be discussed here.

As an example of porous media approaching the idealised capillary model of a porous body we may consider the Anopore membrane with pore size of about 0.2 μm retaining of 100% of latex particles having a size of 0.23 μm, which has capillaries with little tortuosity: $b - a \rightarrow \min$; $k - 1 \rightarrow \min$.

The qualitative conclusions, that the transfer properties of the porous medium are improved with the decrease of system degree of disorder, are supported by the phenomena observed in practice. In particular, it is stated that the hydraulic resistance of the layer consisting of the particles of wide fraction is greater than that of the layer with the same average size of the grain but of narrow fraction.

6.5. Information entropy for numerous parameters

A quantitative characteristic of porous materials isotropy in any desired directions. For this purpose as a measure a totality of capillary pores modelling by uniform distribution upon axis direction of desired coordinate system proposed [17]. In the model pores in different direction are not intersect each other. The idea was through the pore size distribution found average value of pore size upon desired directions and used this value as measure of porous media isotropy.

The information entropy approach may be useful for working out a quantitative characteristic of porous materials isotropy in any desired directions. By means of H it can be made for instance through joint entropy of pore size distribution upon desired directions (for example, Euclidean directions x, y and z):

$$H_r(X,Y,Z) = H_r(X) + H_r(Y) + H_r(Z) = \sum_{i=1}^{3} H_r(i), \quad (23)$$

where $H_r(i)$ is the entropy of pore size distribution upon desired direction i.

Even more, in principle, for independent properties joint entropy H(Joint) is

$$H(\text{Joint}) = \sum_{j=1}^{m} \sum_{i=1}^{n} H_{Pj}(i), \quad (24)$$

where H_{Pj} is the entropy of a property j upon direction I. For instance, property H_{P1} is the pore size, H_{P2} the pore tortuosity, H_{P3} the specific fraction of active area or active centres in pores (catalysis processes), etc. up to H_{Pm}. But it is a special problem that needs careful investigation and experimental testing.

7. Conclusion

The first approach of the method of using information entropy presented in this paper confirms that it is possible to obtain the joint characteristics of porous media and may give a new point of view on the problem of porous media modelling. Industry has a large experience in design and produce porous materials with given properties and the method described above may assist in the analysis of the medium joint characteristics that comprise its different properties.

It is hoped that the quantification of porous media structure irregularity via information entropy value would open the possibility for both experimental and theoretical investigations of links between the "information dimensions" and various transport phenomenon in porous media.

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Appendix A

A.1. The List of some distribution for $x > 0$

(1) *Exponential distribution*

$$f(x) = \lambda \exp(-\lambda x) \quad x \geq 0, \lambda > 0$$

(2) *Gamma distribution*

$$f(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\lambda x) \quad x > 0$$

(3) *Log normal distribution*

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp\left[-\frac{(\ln(x) - m)^2}{2\beta^2}\right] \quad x > 0$$

(4) *Weibull distribution*

$$f(x) = \alpha\lambda x^{\alpha-1} \exp(-\lambda x^{\alpha}) \quad x > 0$$

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