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EFFECT OF TORTUOSITY ON TRANSPORT PROPERTIES OF MIXED GRANULAR BEDS

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Abstract. The properties of binary mixed beds were investigated in a wide range of the particle size ratio δ . The relationship between tortuosity T , porosity ε , δ , and volume fraction of the large particle x_D in mixtures was analysed. Disregarding the tortuosity increase for the range of dense packing led to error in the permeability and diffusivity calculation. The tortuosity variation in the mixed beds must be taken into consideration for modelling transport phenomena in granular mixed beds as well as for building porous media with pre-selected properties. A model of the mixed bed porosity and tortuosity is proposed. The maximum packing tortuosity can be calculated as the product of monosized packing tortuosity both for large and small particle fractions.

Key words: permeability, porosity, tortuosity, mixed granular beds.

The tortuosity of porous media is a complex parameter. It basically consists in the ratio between a substance pathway length and a porous medium thickness. Sometimes constriction or shape of pores must also be considered. Many mass and flow transfer model applications assume the tortuosity T as a constant included in a permeability coefficient $k \propto 1/T^2$ or in an effective diffusion coefficient $D_e \propto 1/T$. However, for example the tortuosity of a spherical particle bed depends on the porosity ε as $T \propto 1/\varepsilon^m$, where $1 > m > 0$ is the empirical value often assumed to be 0.4 – 0.5 for granular beds [1].

The constant tortuosity assumption may be reasonable for monosized granular beds because of the narrow range of the porosity variation between a loose packing ($\varepsilon \approx 0.42$) and a dense ($\varepsilon \approx 0.38$) packing. For instance, the maximum error in the permeability calculation is around 11 – 15% if $m = 0.5$ is assumed. Nevertheless, in mixed granular beds this assumption leads to significant errors because of the wide range of the bed porosity. A large number of experiments were made to confirm the relationship between the tortuosity and the porosity of binary mixed beds. The first set of experiments was made by two-dimension image modelling for particle size ratio δ up to 15 [2]. The second set of experiments used glass beads mixtures with δ up to 10 and permeability determination from which tortuosity was determined. The third set of experiments used mixtures of glass beads and kieselguhr [1]. For all types of mixed beds the dependence of the porosity and of the tortuosity on larger particle volume fraction x_D was obtained. Experimental data show the complicated relation of the tortuosity and porosity with x_D , even in a simplified geometrical model. Both relations, ε and T vs.

x_D have extremes: ε reaches a minimum, whereas T reaches a maximum. Relationships of the tortuosity and x_D are shown in Fig. 1 and depend on the particle size ratio as well as on the particle shape. For glass beads with size ratio 10, the tortuosity exceeds the monosized bed tortuosity T_0 by up to 22%. At mixed glass beads and kieselguhr the ratio of the maximal and a pure kieselguhr bed tortuosity ranges from 1.3 to 1.6. The data obtained on beds permeability k show that the permeability curve profile is minimum in the range of $x_D = 0.45-0.65$ (glass beads) or $0.6 - 0.85$ (kieselguhr + glass beads). For example, the obtained data and its relationship with the permeability on x_D calculated by assuming the tortuosity variation with x_D shown in Fig. 2(a). The mixed bed permeability are now satisfyingly measured and calculated. Disregarding the tortuosity increase in what binary mixtures are concerned led to an error of 20 - 40% in the permeability calculation which depends on δ and particle types.

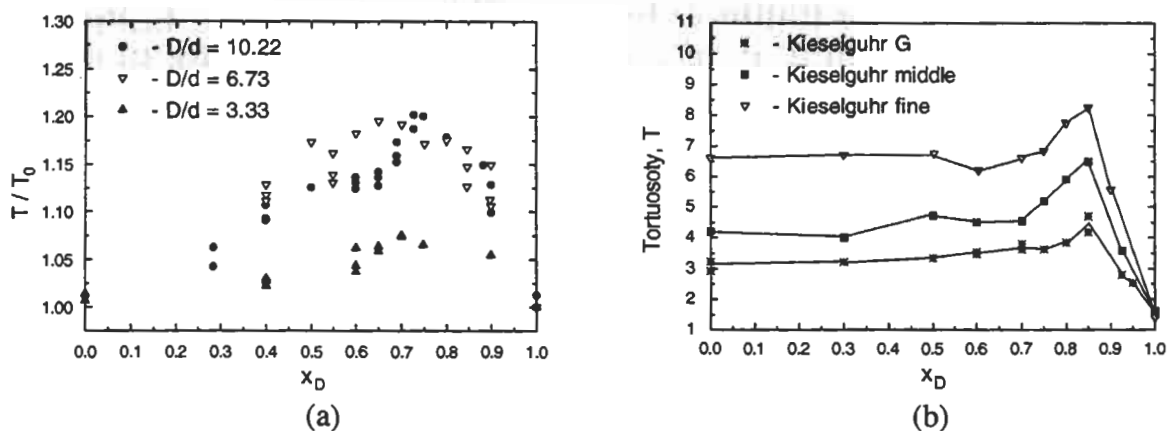


Fig. 1. Dependence of the normalised tortuosity T/T_0 of glass beads mixture (a), and glass beads + kieselguhr bed (b) on x_D , where D and d are the large and small size of the particle fraction in the mixture, respectively. Kieselguhr G is the kieselguhr for thin layer chromatography. Kieselguhr middle and fine are the industrial middle and fine grade kieselguhr, respectively.

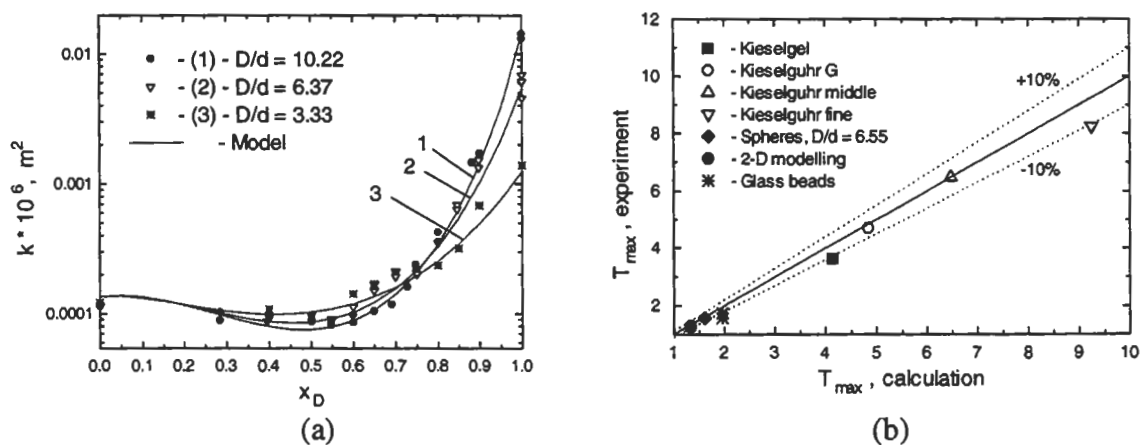


Fig. 2. Relationship: (a) of the glass bed permeability k on x_D , and (b) obtained and predicted maximum tortuosity T_{\max} (b).

The porosity and tortuosity behaviour in mixed beds can be characterised on a particular porosity and tortuosity basis. Let us introduce a particular porosity ε_i as a ratio of residual free space, v_i , after placing i particle fractions into a mixture with a total volume of v_0 . As there is a residual free space v_{i-1} , thus $\varepsilon_i = v_i / v_{i-1}$, where $i = 1, \dots, n$. Hence, v_n corresponds to the final free space in the total mixture volume when all n particle fractions are located in the mixture unit volume. The generalised relation of the unit mixture volume $v_0 = 1$ becomes:

$$\prod_{i=1}^n \varepsilon_i = v_n / v_0 = v_n = \varepsilon$$
 and, in particular, for a binary mixture: $\varepsilon = \varepsilon_D(x_D) \cdot \varepsilon_d(x_D)$. The boundary conditions for the particular porosity of the large $\varepsilon_D(x_D)$ and small $\varepsilon_d(x_D)$ particles size fraction are $\varepsilon_D = 1$, $\varepsilon_d = \varepsilon_d^0$ when $x_D = 0$ (ε_d^0 is the small particle monolayer porosity), and when $x_D = 1$ then $\varepsilon_D = \varepsilon_D^0$, $\varepsilon_d = 1$, (ε_D^0 is the large size particle monolayer porosity). In the mixture there will be a mutual effect between small and large particles. Accordingly, ε_D and ε_d will depend on δ and on the amount of the large particles in the mixture. The minimal mixture porosity ε_{\min} depends mainly on the particle size ratio and for the extreme case of $\delta = d/D \rightarrow 0$ approaches $\varepsilon_{\min}^0 = \varepsilon_D \cdot \varepsilon_d \rightarrow \varepsilon_D^0 \cdot \varepsilon_d^0$.

In the simplified assumption of $T = 1/\varepsilon^m$ it is possible to write the following series
$$T = 1/\varepsilon^m = 1/\prod_{i=1}^n \varepsilon_i = \prod_{i=1}^n (1/\varepsilon_i) = \prod_{i=1}^n T_i$$
, where $T_i = 1/\varepsilon_i$ is the particular tortuosity to which contributed the particle size fraction i into the overall mixture tortuosity T . Respectively, for binary packing $T = T_D \cdot T_d$ and in the extreme case of $\delta \rightarrow 0$ the maximum tortuosity of binary packing approaches $T_{\max}^0 = T_D \cdot T_d \rightarrow T_D^0 \cdot T_d^0$, where T_D^0 and T_d^0 are the large and small particle monolayer tortuosities. In general, the tortuosity maximum does not coincide with the porosity minimum but is located in the same range of x_D and therefore at first sight it is possible to assume that the extreme porosity and tortuosity are at the same x_D .

The tortuosity of the binary bed in the range of maximum tortuosity may be interpreted as a complex of two components. The mixture maximum tortuosity is characterised as a large particle skeleton and the void space is completely filled by smaller particles. The first component of the overall tortuosity represents the tortuosity generated in the bed space by the large particle skeleton T_D . It will be called macro-tortuosity. The second component of T represents the tortuosity T_d of the smaller particles fraction. It may be called micro-tortuosity. When the particle size ratio δ decreases, the smaller particles fraction structure approaches the monosized bed behaviour. This means that for $\delta \rightarrow 0$ it is $T_D \rightarrow T_D^0$, $T_d \rightarrow T_d^0$ and $T_{\max} \rightarrow T_{\max}^0$.

According to the experimentation and the model, the porosity changes are more pronounced than the tortuosity. For a binary mixture of moderate packing density and $\delta = 0.1$,

$\varepsilon_{\min} \approx 0.256 > \varepsilon_{\min}^0$ was obtained, whereas for binary sphere packing with $\varepsilon_D^0 = \varepsilon_d^0 = 0.4$ the global minimum is $\varepsilon_{\min}^0 = 0.16$. For this condition $T_{\max} \approx 1.8$ was obtained when $T_D^0 = T_d^0 = 1.45$, and the global maximum was $T_{\max}^0 = 2.1$. Therefore, with some error, the binary mixture maximum tortuosity can be estimated by means of $T_{\max}^0 = T_D^0 \cdot T_d^0$. The obtained and calculated values of T_{\max} are shown in Fig. 2(b). Based on the data presented above it is possible to use T_{\max}^0 as a preliminary estimation of the tortuosity of binary mixed bed when $\delta \leq 0.1$.

The model needs an improvement to be applicable for full range of particle fraction and particle size ratios. Particle displacement, shape, size ratio, friction force, mode of mixing, etc., affect packing density and, hence the mixed bed porosity and tortuosity. To correlate the model with the influence of mentioned factors a correction function must to be introduced: $\varepsilon = [\varepsilon_D \cdot f(x_D, \delta)] \cdot [\varepsilon_d \cdot f(x_d, \delta)]$.

However, when the relationship of the tortuosity and porosity is still considered to be $T \propto 1/\varepsilon^m$, the overall tortuosity can be obtained directly from the overall porosity.

CONCLUSIONS

Investigation of the porosity and tortuosity of binary granular beds shows the significance of the tortuosity variation with changing the particle volume fractions in the mixture. This fact must be taken into account for the porous media permeability and diffusivity models to avoid transport phenomena miscalculations. The overall porosity and tortuosity will be a multiplying function of each particle fraction.

The model based on the multiplying law gives the possibility to control overall porosity and tortuosity by means of adjusting the particle fractions properties (particle shape and size) and the mixture composition. The model predicts the global minimum porosity and maximum tortuosity which can be achieved for a given mixture composition of particles. Moreover, if the correction functions for the porosity are known, the model may allow to predict the overall porosity and tortuosity in the full range of x_D for different particle size ratios.

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